Algorithms in ParAFEMImp: A Parallel and Wideband Impedance Extraction Program for Complicated 3-D Geometries

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Abstract—In this paper we describe the algorithms used in ParAFEMImp, a parallel program for wideband impedance extraction in very complicated geometries of conductors. Eddy-current equations are solved using an adaptive finite element method. A PDE(Partial Differential Equation) based preconditioner is developed and implemented in the program. Computational results of some large scale adaptive finite element simulations with up to 1 billion degrees of freedom and using up to 1024 CPU cores are presented to demonstrate that ParAFEMImp is robust and scalable for analysis of very complicated geometries of conductors.

I. Introduction

In high-performance integrated circuits, interconnects dominate the performance of the whole system. Accurate estimates of the coupling impedance of these interconnects with complicated three dimensional structures are very important for determining final circuit speeds or functionality. The standard problem in this case consists of the determination of the equivalent parameters in the domains where the full Maxwell's equations or the magneto-quasi-static problem should be solved. There are great efforts in the engineering literature to solve the problem based on the volume integral method [1], [2], [3]. Many electromagnetic analysis tools are developed for wideband analysis of very complicated geometries [2], [5]. Though these programs have been proven very effective, there is no parallel program running in a distributed memory computer or high performance computer which provides with extraordinary computation capability to solve the large scale problems in real circuits.

In this paper, we describe the algorithms used in ParAFEMImp, a parallel program for wideband impedance extraction in very complicated geometries of conductors. A recently developed adaptive finite element method for the circuit/field couplings problem [4] is implemented in the program. And a PDE (Partial Differential Equation) based preconditioner is used to solve the algebraic system resulted from edge element discretization. The parallel implementation is based on MPI (Message Passing Interface) and OpenMP (Open Multi-Processing). The adaptive edge element method is described in the next section, and then the preconditioner is described in section III. The parallel implementation will be detailed in section IV. A variety of numerical examples are examined in section V and the conclusions are given in section VI.

II. The Adaptive Finite Element Method

The adaptive edge element methods based on the *a posteriori error estimates* have the very desirable quasi-optimality property: the energy error decays like $N^{-1/3}$, where N is the number of unknowns.

A. Mathematical Model

The eddy current model with voltage or current excitations in this paper is based on the $\mathbf{A}-\phi$ model in [1] where an integral formulation of the model is developed.

Let Ω be a simply connected bounded domain which contains the conducting region $\Omega_{\rm c}$ and the nonconducting region $\Omega_{\rm nc}=\Omega\backslash\bar{\Omega}_{\rm c}$. The conducting body $\Omega_{\rm c}$ is fed by N external sinusoidal voltage generators through electrodes $S_1,\cdots,S_N$.

Let $\phi_0\in H^1(\Omega)$ be any function that satisfies $\phi_0=U_j$ on S_j, $j=1,\cdots,N$, and $s>0$ be the characteristic size of the domain. $\mathbf{A}$ satisfies

$$\nabla\times\nabla\times\mathbf{A}+\mathbf{i}s^2\sigma\mu\omega\mathbf{A}=-s\sigma\mu\nabla\phi_0+s^2\mu\mathbf{J}_s\quad\text{in }\Omega.$$

The impedance on the electrode S_j is defined as $Z_j = R_j + \mathbf{i}\omega L_j = \frac{U_j}{I_j}$, where U_j is the voltage and I_j is the total current on the electrode S_j

$$I_j = \int_{S_j} \sigma \mathbf{E} \cdot \mathbf{n} ds = \int_{S_j} \sigma(-\mathbf{i}\omega \mathbf{A} - s^{-1}\nabla\phi_0) \cdot \mathbf{n} ds.$$

R_j and L_j are the usual electric resistance and inductance.

B. Finite Element Approach

Let $\mathcal{M}_h$ be a regular tetrahedral mesh of Ω and $\mathbf{U}_h$ be the Nédélec lowest order edge element space [7] over $\mathcal{M}_h$. The finite element approximation is: Find $\mathbf{A}_h \in \mathbf{U}_h$ such that $\forall \mathbf{G}_h \in \mathbf{U}_h$

$$a(\mathbf{A}_h, \mathbf{G}_h) = -s\mu \int_{\Omega_c} \sigma \nabla\phi_0 \mathbf{G}_h dx + s^2\mu \int_{\Omega} \mathbf{J}_s \mathbf{G}_h dx. \quad (1)$$

where,

$$a(\mathbf{A}_h, \mathbf{G}_h) = \int_{\Omega} \mathbf{curl}\mathbf{A}_h \cdot \mathbf{curl}\mathbf{G}_h dx + \mathbf{i}s^2\mu\omega \int_{\Omega} \sigma \mathbf{A}_h \mathbf{G}_h dx$$

C. A Posteriori Error Estimates

A posteriori error estimates are computable quantities in terms of the discrete solution and known data that measure the actual discrete errors without any knowledge of exact solutions. The *a posteriori error estimator* η_T^2 on element T used in this paper is defined in our other paper [4].

D. Adaptive Algorithm

The adaptive finite element method based on *a posteriori error estimates* is characterized by the *solve→estimate→mark→refine* loop.

III. PRECONDITIONER

The problem (1) which results in a singular algebraic system of equations can be solved by a preconditioned GMRES method. The preconditioning matrix corresponds to the finite element discretization of the following problem:

$$\nabla \times \nabla \times \mathbf{A} + s^2\omega\sigma\mu\mathbf{A} = \mathbf{f} \text{ in } \Omega. \quad (2)$$

The preconditioning problem can be efficiently solved by a PCG method using the AMS (Auxiliary space Maxwell Solver) preconditioner.

IV. PARALLEL IMPLEMENTATION

The dynamic load balancing is based on the partition of mesh.

V. NUMERICAL RESULTS

In this section, we firstly consider the parasitic parameters of a straight conductor. Fig. 1 shows the relation between the resistance and the frequency. We compare the results computed by the FastImp and our program ParAFEMImp. The accuracy of ParAFEMImp is comparable with FastImp's.

Then, numerical results for two real circuits (inverter circuit and adder circuit) will be given. The computations with up to 1 billion unknowns were performed on the cluster LSSC-III in the State Key Laboratory on Scientific and Engineering Computing of Chinese Academy of Sciences, using up to 1024 CPU cores. The stable iteration numbers of the preconditioned GMRES indicate that the preconditioner is optimal, see table I. The numerical results demonstrate that ParAFEMImp is accurate, robust and scalable for electromagnetic analysis of very complicated geometries of conductors.

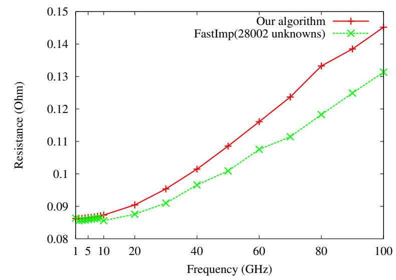

Fig. 1. The relation between the resistance and the frequency (Line Conductor)

TABLE I
THE NUMBER OF GMRES ITERATIONS IN CASE OF DIFFERENT FREQUENCES (ADDER CIRCUIT, 1024 CPU CORES).

frequence	# of unknowns	nits	# of unknowns	nits
1GHz	799424370	11	1119245084	13
10Ghz	740450796	13	1037479176	13
100Ghz	744519670	15	1053917232	10

VI. CONCLUSION

We have developed a parallel impedance extraction program, ParAFEMImp, for large scale simulations. Numerical examples show that ParAFEMImp is robust and scalable. It means that the computing time can be significantly reduced by using the massively parallel computers.

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